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# A 10-form Gauge Potential and an M-9-brane Wess-Zumino Action in Massive 11D Theory

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## Abstract

We discuss some properties of an M-9-brane in “massive 11D theory” proposed by Bergshoeff, Lozano and Ortin. A 10-form gauge potential is consistently introduced into the massive 11D supergravity, and an M-9-brane Wess-Zumino action is constructed as that of a gauged  $\sigma$ -model. Using duality relations is crucial in deriving the action, which we learn from the study of a 9-form potential in 10D massive IIA theory. A target space solution of an M-9-brane with a non-vanishing 10-form gauge field is also obtained, whose source is shown to be the M-9-brane effective action.

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# 1 Introduction

M-theory is conjectured to be the strong coupling limit of the 10 dimensional (10D) type IIA string theory, and the 10D type IIA string theory is to be the  $S^1$  compactified M-theory in the vanishing limit of the radius[1]. So, based on this conjecture, the 10D type IIA theory and all of its constituents should have their 11-dimensional (11D) origins[2][1]. As is well known, however, the 11D origin of 10D massive IIA theory has not been understood yet. (The 10D massive IIA theory refers to the 10D IIA string theory with a nonzero RR 10-form field strength[3] and its field theory limit, 10D massive IIA supergravity[4][5].) This is due to the fact that the massive IIA supergravity has a cosmological term composed of a mass parameter  $m$ . Since there is the no-go theorem that 11D supergravity forbids a cosmological term[6], the term (and hence the mass) cannot be derived from the 11D supergravity via dimensional reduction. The mass is the dual of the field strength of a RR 9-form potential to which a D-8-brane couples. So, the 11D origin of the D-8-brane is also unclear, although it is conjectured to be an M-9-brane from studies of the M-theory superalgebra[7][8]. Some attempts have been made[9][10][11][12], and one of them is “massive 11-dimensional (11D) theory” proposed by Bergshoeff, Lozano, and Ortin[10][11]. We investigate this massive theory in this paper.

First, we give a brief review of the massive 11D theory. In this theory, target-space fields are required to have a Killing isometry (whose direction is parameterized by  $z$ ). Its field theory, called “massive 11D supergravity”[10], gives the bosonic part of the 10D massive IIA one upon dimensional reduction in  $z$ . Though this is a bosonic theory, it is called “supergravity” since it reproduces the bosonic part of the ordinary 11D supergravity in the massless limit (and if the dependence of the fields on  $z$  is restored). Target-space solutions of M-9-branes are also obtained[13], which give D-8-brane solutions[14][5] when dimensionally reduced over  $z$ . In addition, worldvolume actions of M-branes in this supergravity background have also been examined. In the cases of an M-wave, an M-2-brane, an M-5-brane and an M-Kaluza-Klein monopole, full worldvolume actions are obtained as those of gauged sigma models[10][11]. These actions also give those of the 10D massive IIA branes suggested by the 10D IIA superalgebra upon direct or double dimensional reduction.<sup>†</sup> In the case of an M-9-brane, however, *a Wess-Zumino (WZ) action has not been constructed yet*, although a kinetic term of the action has been constructed [13][15].<sup>‡</sup>

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<sup>†</sup>Furthermore, actions of branes *not* predicted by the IIA superalgebra are also obtained[15][16].

<sup>‡</sup>It is considered that an M-9-brane cannot be singled out in 11 uncompactified dimensions, but it can be singled out when there is one compact dimension[17]. Based on the discussion on how the M-9-brane tension scales with the radius of the 11th compact dimension[13], the action in this approach describes

The main purpose of this paper is to construct the WZ action of an M-9-brane in a massive 11D supergravity background, since WZ actions of branes also play important roles in investigating properties of branes and dualities (e.g. see the recent paper [18]). For this purpose, a 10-form gauge potential is needed since a p-brane naturally couples to the (p+1)-form potential. So, specifically, we introduce the 10-form potential into the massive 11D supergravity and construct the M-9-brane WZ action by using the 10-form.

In introducing a 10-form potential, we follow the case of a 9-form gauge potential in the 10D massive IIA supergravity. There are two methods to introduce the 9-form: The first one is to promote the mass parameter  $m$  to a scalar field  $M(x)$  and to introduce a 9-form potential as a Lagrange multiplier for the constraint  $dM(x) = 0$ [5]. (We denote this 9-form as  $A^{(9)}$ .) The second one is superspace formulation where the basis of the field variables of RR (p+1)-form is inspired by the coupling to D-p-branes[19][20].<sup>§</sup> In this case the gauge transformations and the field strengths of dual RR gauge fields are suggested on the basis that the RR gauge fields can be dealt with in a geometrically uniform way, and their consistency is checked by discussing T-dualities[20]. The field strength of the 9-form potential is defined as a dual of the mass parameter  $m$ . (We denote this 9-form as  $C^{(9)}$ .) In this paper we choose the first one to introduce a 10-form potential into the 11D theory. The reason for this choice is as follows: In the first method the massive gauge transformation of the 10-form potential is automatically determined. On the other hand, if one applies the second method to the 11D case, one have to construct by hand a massive gauge transformation and a field strength of the 10-form consistently, which seems difficult since there is no geometric uniformity or T-duality symmetry in the massive 11D case.

However, *an gauge invariant M-9-brane WZ action cannot be constructed by using the 10-form at least straightforwardly.* In fact this is also the case with the D-8-brane WZ action  $S_{D8}^{WZ}$  and the 9-form  $A^{(9)}$  in 10D IIA theory,<sup>¶</sup> where the 9-form used to construct  $S_{D8}^{WZ}$  is  $C^{(9)}$ . (That is, the field redefinition relating the two 9-forms has not been found.) So, first of all, we explore the way to construct  $S_{D8}^{WZ}$  in terms of  $A^{(9)}$ . As a result, we show that *a gauge invariant D-8-brane WZ action can be constructed in terms of  $A^{(9)}$  if duality relations are appropriately used* (to be concrete, for rewriting the expression of the massive gauge transformation of  $A^{(9)}$ ). So, based on this lesson, we repeat the same procedures in the massive 11D theory: We discuss field strengths of gauge fields and their duals and assume their duality relations, based on their transformations properties and their relations to 10D IIA fields. Then, we use the relations appropriately to rewrite the

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an M-9-brane wrapped around the compact isometry direction.

<sup>§</sup>The requirement of  $\kappa$ -symmetry of the D-brane actions is shown to imply the field equations of massive IIA supergravity[21][22].

<sup>¶</sup>Detailed discussions are given in section 2.

massive gauge transformation of  $\hat{A}^{(10)}$ . Finally, we construct a gauge invariant M-9-brane WZ action which gives the D-8-brane WZ action on dimensional reduction along  $z$ .

Moreover, this paper has another purpose: to reconstruct the target-space solution of a single M-9-brane given in ref.[13]. Two points are improved: The first one is that we solve the equations of motion (of the massive 11D supergravity) *with the source terms*, which comes from the obtained WZ action as well as the kinetic action of an M-9-brane. The source terms have not been taken into account in ref.[13], but they must be considered since an M-9-brane can be regarded roughly as an electric object in terms of the 10-form gauge field. The second one is that we construct an M-9-brane solution *with a nontrivial configuration of 10-form potential*. The solutions in ref.[13] are obtained as solutions of a *pure* gravity (i.e. only the metric field is nontrivial). However, this seems unnatural because usual p-brane solutions are obtained as those with nontrivial (p+1)-form gauge potentials. To be concrete, we make a certain ansatz, including the one done in ref.[13], and solve the equations of motion with the source terms.

The organization of this paper is as follows: In section 2 we discuss the case of the 9-form in 10D massive IIA supergravity. We first give a review of it, and then exhibit the problem and its resolution stated above explicitly. In section 3, after a short review of the massive 11D supergravity, we introduce a 10-form gauge potential. Then, we discuss duality relations, use them appropriately and construct an M-9-brane WZ action. In section 4 we solve the equations of motion with source terms and present an M-9-brane solution with a nontrivial 10-form potential. In section 5 we give short summary and discussion. In the appendix, we give the relations between the 11D and the 10D fields.

## 2 A 9-form potential in 10D massive IIA theory

In this section we begin with a brief review of the 10D massive IIA supergravity[4][5]. It has the same field content as the massless one:  $\{g_{\mu\nu}, B_{\mu\nu}, \phi, C_\mu^{(1)}, C_{\mu\nu\rho}^{(3)}\}$ .<sup>||</sup> The (infinitesimal) massive gauge transformations are defined as

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\lambda_{\nu]}, \quad \delta C_\mu^{(1)} = -m\lambda_\mu, \quad \delta C_{\mu\nu\rho}^{(3)} = -3mB_{[\mu\nu}\lambda_{\rho]} \quad (2.1)$$

where  $\lambda_\mu$  is a 1-form gauge parameter and  $m$  is a constant mass parameter. The gauge invariant field strengths of them are

$$\begin{aligned} H_{\mu\nu\rho}^{(3)} &= 3\partial_{[\mu}B_{\nu\rho]}, & G_{\mu\nu}^{(2)} &= 2\partial_{[\mu}C_{\nu]}^{(1)} + mB_{\mu\nu}, \\ G_{\mu\nu\rho\sigma}^{(4)} &= 4\partial_{[\mu}C_{\nu\rho\sigma]}^{(3)} - 12\partial_{[\mu}B_{\nu\rho}C_{\sigma]}^{(1)} + 3mB_{[\mu\nu}B_{\rho\sigma]}. \end{aligned} \quad (2.2)$$

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<sup>||</sup>We use mostly minus metric for both target-spaces and worldvolumes. Target-space fields with hats are 11-dimensional, and those with no hats are 10 dimensional.

The bosonic action of the massive IIA supergravity is \*\*

$$\begin{aligned}
S_0 = & \frac{1}{16\pi G_N^{(10)}} \int d^{10}x [\sqrt{|g|} \{ e^{-2\phi} (R - 4(\partial\phi)^2 + \frac{1}{2 \cdot 3!} (H^{(3)})^2) \\
& - \frac{1}{4} (G^{(2)})^2 - \frac{1}{2 \cdot 4!} (G^{(4)})^2 + \frac{1}{2} m^2 \} \\
& + \frac{1}{144} \epsilon^{\mu_1 \dots \mu_{10}} \{ \partial C^{(3)} \partial C^{(3)} B + \frac{1}{2} m \partial C^{(3)} (B)^3 + \frac{9}{80} m^2 (B)^5 \}_{\mu_1 \dots \mu_{10}}] \quad (2.3)
\end{aligned}$$

where  $\epsilon$  is the totally antisymmetric symbol ( $\epsilon^{012\dots 9} = 1$ ). The bosonic action of the 10D massless IIA supergravity can be found by taking the limit  $m \rightarrow 0$ .

One way to introduce a 9-form gauge potential is to promote the mass parameter  $m$  to a scalar field  $M(x)$ , and to add the term

$$\Delta S = \frac{1}{16\pi G_N^{(10)}} \int d^{10}x \frac{1}{10!} \epsilon^{\mu_1 \dots \mu_{10}} M(x) 10 \partial_{[\mu_1} A_{\mu_2 \dots \mu_{10}] }^{(9)} \quad (2.4)$$

to the action  $S_0[5]$ . Then, the field equation of  $A^{(9)}$  implies that the scalar field  $M(x)$  is a constant  $m$ ,<sup>††</sup> So, eliminating  $A^{(9)}$  leads to the field equations of the original massive IIA supergravity. After the above procedure, the  $M(x)$  field equation is

$$\begin{aligned}
-M = & \frac{\epsilon^{\mu_1 \dots \mu_{10}}}{\sqrt{|g|}} \{ \frac{10}{10!} \partial A^{(9)} + \frac{1}{288} \partial C^{(3)} B^3 + M \frac{9}{144 \cdot 40} B^5 \}_{\mu_1 \dots \mu_{10}} \\
& - \frac{1}{2} G^{(2)\mu\nu} B_{\mu\nu} - \frac{1}{8} G^{(4)\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma}, \quad (2.5)
\end{aligned}$$

implying that the 10-form field strength  $F^{(10)} = 10 \partial A^{(9)}$  is regarded as the variable canonically conjugate to  $M$ .

At this moment, the original action  $S_0$  is no longer invariant under the transformations (2.1). Instead,  $\delta S_0$  is proportional to  $\partial M$ , as

$$\begin{aligned}
\delta S_0 = & \frac{1}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|} [\partial_\mu M \{ G^{(2)\mu\nu} \lambda_\nu + \frac{1}{2} G^{(4)\mu\nu\rho\sigma} B_{\nu\rho} \lambda_\sigma \} \\
& - \frac{\epsilon^{\mu_1 \dots \mu_{10}}}{\sqrt{|g|}} \partial_{\mu_1} M \{ \frac{1}{48} \partial C^{(3)} B^2 \lambda + \frac{M}{192} B^4 \lambda \}_{\mu_2 \dots \mu_{10}}]. \quad (2.6)
\end{aligned}$$

This variation can be cancelled (up to total derivative) by defining such a variation of the 9-form gauge potential  $A^{(9)}$  as

$$\begin{aligned}
\delta A_{\mu_1 \dots \mu_9}^{(9)} = & \sqrt{|g|} \epsilon_{\mu_1 \dots \mu_9 \mu} \{ G^{(2)\mu\nu} \lambda_\nu + \frac{1}{2} G^{(4)\mu\nu\rho\sigma} B_{\nu\rho} \lambda_\sigma \} \\
& - 9! \{ \frac{1}{48} \partial C^{(3)} B^2 \lambda + \frac{M}{192} B^4 \lambda \}_{[\mu_1 \dots \mu_9]}. \quad (2.7)
\end{aligned}$$

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\*\* As discussed in ref.[13], the sign of the cosmological constant is determined by T-duality.

†† Strictly speaking,  $M(x)$  is piecewise constant. We discuss this point in section 4.

So, it is concluded that the 9-form potential is introduced consistently.<sup>‡‡</sup> We note that the first two terms of (2.7) cannot be expressed as exterior products of forms.

Now, we discuss the construction of the D-8-brane WZ action  $S_{D8}^{WZ}$ . What we want to do is to construct  $S_{D8}^{WZ}$  by using the 9-form  $A^{(9)}$ . However, this cannot be achieved at least straightforwardly. To be concrete, since a D-8-brane couples to a 9-form potential,  $S_{D8}^{WZ}$  contains the term

$$S_{D8}^{WZ}|_{9\text{-form part}} = \frac{T}{9!} \int d^9 \xi \epsilon^{i_1 \dots i_9} \partial_{i_1} X^{\mu_1} \dots \partial_{i_9} X^{\mu_9} A_{\mu_1 \dots \mu_9}^{(9)} \quad (2.8)$$

where  $\xi_i$  ( $i = 0, \dots, 8$ ) are worldvolume coordinates of the brane and  $X^\mu$  ( $\mu = 0, \dots, 9$ ) are embedding coordinates. Suppose we consider the massive gauge transformation of (2.8). Then, we can see that the contribution of the first two terms of (2.7) to the variation of  $S_{D8}^{WZ}|_{9\text{-form part}}$  cannot be cancelled even if any other terms are added to  $S_{D8}^{WZ}|_{9\text{-form part}}$ . This implies that one cannot keep the massive gauge symmetry of the brane action if  $A^{(9)}$  is used straightforwardly.

Our idea to resolve this problem is very simple: if one rewrite the first two terms of (2.7) by using the dual fields of  $C^{(1)}$  and  $C^{(3)}$  through duality relations, the two terms can be expressed as a sum of exterior products of forms. So, it is expected that a gauge invariant WZ action can be constructed.

This idea is a success, which we show in the following: The dual fields of  $C^{(3)}$  and  $C^{(1)}$  is the 5-form  $C^{(5)}$  and the 7-form  $C^{(7)}$ , respectively. We use the duals defined in ref.[20], whose field strengths and massive gauge transformations are given respectively as

$$\begin{cases} G_{\mu_1 \dots \mu_6}^{(6)} = (6\partial C^{(5)} - 60\partial B C^{(3)} + 15MB^3)_{[\mu_1 \dots \mu_6]}, \\ G_{\mu_1 \dots \mu_8}^{(8)} = (8\partial C^{(7)} - 168\partial B C^{(5)} + 105MB^4)_{[\mu_1 \dots \mu_8]}, \end{cases} \quad \begin{cases} \delta C_{\mu_1 \dots \mu_5}^{(5)} = -15(B^2 \lambda)_{[\mu_1 \dots \mu_5]}, \\ \delta C_{\mu_1 \dots \mu_7}^{(7)} = -105(B^3 \lambda)_{[\mu_1 \dots \mu_7]}. \end{cases}$$

The duality relations of them (in our notation) are[20]

$$G^{(2)\mu_1 \mu_2} = -\frac{\epsilon^{\mu_1 \dots \mu_{10}}}{8! \sqrt{|g|}} G_{\mu_3 \dots \mu_{10}}^{(8)}, \quad G^{(4)\mu_1 \dots \mu_4} = \frac{\epsilon^{\mu_1 \dots \mu_{10}}}{6! \sqrt{|g|}} G_{\mu_5 \dots \mu_{10}}^{(6)}. \quad (2.9)$$

Substituting these relations for the first two terms of (2.7) leads to the rewritten expression

$$\delta A_{\mu_1 \dots \mu_9}^{(9)} = -9! \left[ \frac{1}{7!} \partial C^{(7)} \lambda - \frac{1}{2 \cdot 5!} \partial(C^{(5)} B) \lambda + \frac{1}{2^3 \cdot 3!} \partial(C^{(3)} B^2) \lambda - \frac{M}{2^4 \cdot 4!} B^4 \lambda \right]_{[\mu_1 \dots \mu_9]}. \quad (2.10)$$

Using this expression, we can obtain the WZ action invariant under the massive gauge transformations. This means that via a field redefinition,  $A^{(9)}$  is related to the 9-form potential  $C^{(9)}$  which is usually used for constructing  $S_{D8}^{WZ}$ . So, instead of giving the action

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<sup>‡‡</sup>After the procedure, the field strengths of RR gauge fields are also not invariant under the massive gauge transformations due to the promotion of  $m$  to  $M(x)$ . However, this causes no problem because one can replace at any time  $M(x)$  for a constant  $m$  by solving the  $A^{(9)}$  field equation.

in terms of  $A^{(9)}$ , we present the redefinition relation. The field strength of  $C^{(9)}$  is defined in this case as[20]

$$-M = *G^{(10)} = \frac{\epsilon^{\mu_1 \dots \mu_{10}}}{10! \sqrt{|g|}} 10 [\partial C^{(9)} - 36 \partial B \partial C^{(7)} + \frac{189M}{2} B^5]_{\mu_1 \dots \mu_{10}}. \quad (2.11)$$

Substituting (2.9) for the last two terms of (2.5) and comparing it with (2.11), we can determine the relation between the two 9-forms up to total derivative as

$$C^{(9)}_{\mu_1 \dots \mu_9} = \{A^{(9)} + 9! (\frac{1}{2 \cdot 7!} C^{(7)} B - \frac{1}{2^3 \cdot 5!} C^{(5)} B^2 + \frac{1}{2^3 (3!)^2} C^{(3)} B^3)\}_{[\mu_1 \dots \mu_9]}. \quad (2.12)$$

This redefinition relation is consistent with the massive gauge transformations of the two, (i.e. (2.10) and  $\delta C^{(9)}_{\mu_1 \dots \mu_9} = -945M(B^4 \lambda)_{[\mu_1 \dots \mu_9]}$ ). So,  $S_{D8}^{WZ}$  can be constructed via  $A^{(9)}$ . Thus, we conclude that *using duality relations are crucial to derive the the WZ action in terms of the 9-form  $A^{(9)}$ .*

### 3 A 10-form gauge potential and an M-9-brane Wess-Zumino action in massive 11D supergravity

In this section we first review the massive 11D supergravity[10]. The bosonic field content of the supergravity is the same as that of the usual (massless) 11D supergravity: the metric  $\hat{g}_{\mu\nu}$  and a 3-form gauge potential  $\hat{C}_{\mu\nu\rho}$ . In this theory these fields are required to have a Killing isometry, i.e.,  $\mathcal{L}_{\hat{k}} \hat{g}_{\mu\nu} = \mathcal{L}_{\hat{k}} \hat{C}_{\mu\nu\rho} = 0$  where  $\mathcal{L}_{\hat{k}}$  indicates a Lie derivative with respect to a Killing vector field  $\hat{k}^\mu$ . (We take the coordinates so that  $\hat{k}^\mu = \delta^{\mu z}$ .) The infinitesimal gauge transformations of the fields are defined as

$$\delta \hat{g}_{\mu\nu} = -m [\hat{\lambda}_\mu (i_{\hat{k}} \hat{g})_\nu + \hat{\lambda}_\nu (i_{\hat{k}} \hat{g})_\mu], \quad \delta \hat{C}_{\mu\nu\rho} = 3 \partial_{[\mu} \hat{\chi}_{\nu\rho]} - 3m \hat{\lambda}_{[\mu} (i_{\hat{k}} \hat{C})_{\nu\rho]} \quad (3.1)$$

where  $(i_{\hat{k}} T^{(r)}_{\mu_1 \dots \mu_{r-1} \mu}) \equiv \hat{k}^\mu T^{(r)}_{\mu_1 \dots \mu_{r-1} \mu}$  for a field  $T^{(r)}$ .  $\hat{\chi}$  is the infinitesimal 2-form gauge parameter, and  $\hat{\lambda}$  is defined as  $\hat{\lambda}_\mu \equiv (i_{\hat{k}} \hat{\chi})_\mu$ .<sup>\*</sup> Then, a connection for the massive gauge transformations should be considered. The new total connection takes the form  $\hat{\Omega}_a^{bc} = \hat{\omega}_a^{bc} + \hat{K}_a^{bc\dagger}$  where  $\hat{\omega}_a^{bc}$  is a usual spin connection and  $\hat{K}$  is given by

$$\hat{K}_a^{bc} = \frac{m}{2} [\hat{k}_a (i_{\hat{k}} \hat{C})^{bc} + \hat{k}_b (i_{\hat{k}} \hat{C})^{ac} - \hat{k}_c (i_{\hat{k}} \hat{C})^{ab}]. \quad (3.2)$$

The 4-form field strength  $\hat{G}^{(4)}$  of  $\hat{C}$  is defined as

$$\hat{G}^{(4)}_{\mu\nu\rho\sigma} = 4D_{[\mu} \hat{C}_{\nu\rho\sigma]} \equiv 4\partial_{[\mu} \hat{C}_{\nu\rho\sigma]} + 3m (i_{\hat{k}} \hat{C})_{[\mu\nu} (i_{\hat{k}} \hat{C})_{\rho\sigma]} \quad (3.3)$$

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<sup>\*</sup>In this paper we change the notation of ref.[10] such that  $m \rightarrow 2m$  and  $\hat{\lambda} \rightarrow -\frac{1}{2}\hat{\lambda}$ .

<sup>†</sup>We use  $a, b, \dots$  for local Lorentz indices.

where  $D_\mu$  denotes the covariant derivative. Then,  $\hat{G}^{(4)}$  transforms covariantly as

$$\delta \hat{G}_{\mu\nu\rho\sigma}^{(4)} = 4m\hat{\lambda}_{[\mu}(i_{\hat{k}}G^{(4)})_{\nu\rho\sigma]}, \quad (3.4)$$

which implies that  $\delta(\hat{G}^{(4)})^2 = 0$ .

The action of the massive 11D supergravity is

$$\begin{aligned} \hat{S}_0 &= \frac{1}{\kappa} \int d^{11}x [\sqrt{|\hat{g}|} \{ \hat{R} - \frac{1}{2 \cdot 4!} (\hat{G}^{(4)})^2 + \frac{1}{2} m^2 |\hat{k}|^4 \} \\ &+ \frac{\hat{\epsilon}^{\mu_1 \dots \mu_{11}}}{(144)^2} \{ 2^4 \partial \hat{C} \partial \hat{C} \hat{C} + 18m \partial \hat{C} \hat{C} (i_{\hat{k}} \hat{C})^2 + \frac{3^3}{5} m^2 \hat{C} (i_{\hat{k}} \hat{C})^4 \}_{\mu_1 \dots \mu_{11}} ] \end{aligned} \quad (3.5)$$

where  $\kappa = 16\pi G_N^{(11)}$  and  $|\hat{k}| = \sqrt{-\hat{k}^\mu \hat{k}^\nu \hat{g}_{\mu\nu}}$ . This action is invariant (up to total derivative) under (3.1). The dimensional reduction of the action along  $z$  is shown to give the bosonic part of 10D massive IIA supergravity.<sup>‡</sup> (See the appendix for the relation between the 11D and 10D fields.)

Now, let us introduce a 10-form gauge potential  $\hat{A}^{(10)}$ . Following the case of the 9-form potential in 10D IIA theory, we promote the mass parameter  $m$  to a scalar field  $\hat{M}(x)$ , and add the term

$$\Delta S = \frac{1}{\kappa} \int d^{11}x \frac{1}{11!} \hat{\epsilon}^{\mu_1 \dots \mu_{11}} \hat{M}(x) 11 \partial_{[\mu_1} \hat{A}_{\mu_2 \dots \mu_{11}]}. \quad (3.6)$$

We note that  $\hat{A}^{(10)}$  also satisfies  $\mathcal{L}_{\hat{k}} \hat{A}^{(10)} = 0$ , which means that  $\hat{A}^{(10)}$  with no  $z$  index does not appear in this theory. Then, the action is invariant under (3.1) if the massive gauge transformation of  $\hat{A}^{(10)}$  is defined as

$$\begin{aligned} \delta(i_{\hat{k}} \hat{A}^{(10)})_{\mu_1 \dots \mu_9} &= -\sqrt{|\hat{g}|} \hat{\epsilon}_{\mu_1 \dots \mu_9 \mu z} [-\hat{g}^{\mu\mu'} \hat{g}^{\nu\nu'} (2\partial_{[\mu'} \hat{k}_{\nu']} - \hat{M} |\hat{k}|^2 (i_{\hat{k}} \hat{C})_{\mu'\nu'}) \hat{\lambda}_\nu \\ &+ \frac{1}{2} \hat{G}^{(4)\mu\nu\rho\sigma} (i_{\hat{k}} \hat{C})_{\nu\rho} \hat{\lambda}_\sigma] - \frac{9!}{48} [\partial \hat{C} (i_{\hat{k}} \hat{C})^2 \hat{\lambda} + \frac{\hat{M}}{4} (i_{\hat{k}} \hat{C})^4 \hat{\lambda}]_{\mu_1 \dots \mu_9} \end{aligned} \quad (3.7)$$

$$\delta \hat{A}_{\mu_1 \dots \mu_{10}}^{(10)} = 10 \hat{M} \hat{\lambda}_{[\mu_1} (i_{\hat{k}} \hat{A}^{(10)})_{\mu_2 \dots \mu_{10}]} \quad \text{when } \mu_1, \dots, \mu_{10} \neq z \quad (3.8)$$

where (3.8) is the expected massive gauge transformation of a 10-form gauge field.

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<sup>‡</sup>By using the (generalized) Palatini's identity given in ref.[10], the action (3.5) is rewritten as

$$\begin{aligned} \hat{S}_0 &= \frac{1}{\kappa} \int d^{11}x [\sqrt{|\hat{g}|} \{ -\hat{\Omega}_b^{ba} \hat{\Omega}_c^c{}_a - \hat{\Omega}_a^{bc} \hat{\Omega}_{bc}^a - \frac{1}{2 \cdot 4!} (\hat{G}^{(4)})^2 + \frac{1}{2} m^2 |\hat{k}|^2 \} \\ &+ \frac{1}{144} \hat{\epsilon}^{\mu_1 \dots \mu_{10} z} \{ \partial \hat{C} \partial \hat{C} (i_{\hat{k}} \hat{C}) + \frac{m}{2} \partial \hat{C} (i_{\hat{k}} \hat{C})^3 + \frac{9m^2}{80} (i_{\hat{k}} \hat{C})^5 \}_{\mu_1 \dots \mu_{10} z} + (\text{surface terms})]. \end{aligned}$$

In fact, the 10D action (2.3) is obtained from this action only if the surface terms are omitted. Omitting them, we use this action as a “starting” action, in order to make the correspondence of the 11D theory with the 10D one.



Next, we prepare to rewrite the first two terms of the massive transformation (3.7), in the same way as the 10D case. The dual field of the 3-form  $\hat{C}$  is the 6-form  $\hat{C}^{(6)}$  whose massive gauge transformation, field strength and the duality relation are<sup>[10]</sup><sup>§</sup>

$$\delta \hat{C}_{\mu_1 \dots \mu_6}^{(6)} = 30 \partial_{[\mu_1} \hat{\chi}_{\mu_2 \mu_3} \hat{C}_{\mu_4 \mu_5 \mu_6]} + 6 \hat{M} \hat{\lambda}_{[\mu_1} (i_{\hat{k}} \hat{C}^{(6)})_{\mu_2 \dots \mu_6]} \quad (3.9)$$

$$\hat{G}_{\mu_1 \dots \mu_7}^{(7)} = 7 [\partial \hat{C}^{(6)} - 3 \hat{M} (i_{\hat{k}} \hat{C}) (i_{\hat{k}} C^{(6)}) + 10 \hat{C} \partial \hat{C} + 5 \hat{M} C (i_{\hat{k}} \hat{C})^2 + \frac{\hat{M}}{7} (i_{\hat{k}} \hat{N}^{(8)})]_{\mu_1 \dots \mu_7} \quad (3.10)$$

$$\hat{G}^{(4)\mu_1 \dots \mu_4} = \frac{\epsilon^{\mu_1 \dots \mu_{11}}}{7! \sqrt{|g|}} \hat{G}_{\mu_5 \dots \mu_{11}}^{(7)}. \quad (3.11)$$

$\hat{N}^{(8)}$  is the dual field of the Killing vector also introduced in ref.[10], whose gauge transformation is suggested such as

$$\delta \hat{N}_{\mu_1 \dots \mu_8}^{(8)} = \left\{ \frac{8!}{3 \cdot 4!} \partial \hat{\chi} \hat{C} (i_{\hat{k}} \hat{C}) + 8 \hat{M} \hat{\lambda} (i_{\hat{k}} \hat{N}^{(8)}) \right\}_{\mu_1 \dots \mu_8}. \quad (3.12)$$

In this paper we regard  $\hat{k}_\mu \equiv (i_{\hat{k}} \hat{g})_\mu$  as a “vector gauge field”, and consider the “field strength” of it, as done for  $(i_{\hat{k}} \hat{C})$ . Then, if we define  $\hat{G}^{(2)}$  as

$$\hat{G}_{\mu\nu}^{(2)} \equiv 2 \partial_{[\mu} \hat{k}_{\nu]} - \hat{M} |k|^2 (i_{\hat{k}} \hat{C})_{\mu\nu}, \quad (3.13)$$

$\hat{G}^{(2)}$  is shown to transform covariantly under (3.1). So,  $\hat{G}^{(2)}$ , in fact arising in the first term of (3.7), can be interpreted as the field strength of  $\hat{k}_\mu$ . On the other hand, the field strength  $\hat{G}^{(9)}$  of the full 8-form  $\hat{N}^{(8)}$  is difficult to construct. However, in order to rewrite the first term through the duality relation between  $\hat{G}^{(9)}$  and  $\hat{G}^{(2)}$ , it is sufficient to know the field strength of  $(i_{\hat{k}} \hat{N}^{(8)})$ . This is because  $\hat{G}^{(2)}$  in the first term of (3.8) vanishes if any of the indices of  $\hat{G}^{(2)}$  takes  $z$ , implying that one of the indices of  $\hat{G}^{(9)}$  certainly takes  $z$ . Thus, only the field strength of  $(i_{\hat{k}} \hat{N}^{(8)})$  is needed. It can be defined as

$$\begin{aligned} (i_{\hat{k}} \hat{G}^{(9)})_{\mu_1 \dots \mu_8} &\equiv 8 \{ \partial (i_{\hat{k}} \hat{N}^{(8)}) + 21 (i_{\hat{k}} \hat{C}^{(6)}) \partial (i_{\hat{k}} \hat{C}) \\ &+ 35 C \partial (i_{\hat{k}} \hat{C}) (i_{\hat{k}} \hat{C}) + 35 \partial C (i_{\hat{k}} \hat{C})^2 + \frac{105}{8} \hat{M} (i_{\hat{k}} \hat{C})^4 \}_{\mu_1 \dots \mu_8}. \end{aligned} \quad (3.14)$$

We note that  $(i_{\hat{k}} \hat{G}^{(9)})$  is invariant under (3.1), which means that this definition is consistent. Then, we assume the duality relation:

$$\hat{G}^{(2)\mu_1 \mu_2} = \frac{\epsilon^{\mu_1 \dots \mu_{10} z}}{9! \sqrt{|g|}} (i_{\hat{k}} \hat{G}^{(9)})_{\mu_3 \dots \mu_{10}}. \quad (3.15)$$

It gives one of the 10D IIA duality relations in (2.9) on dimensional reduction in  $z$ , which means that (3.15) is consistent.

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<sup>§</sup>We concentrate our discussions on the gauge transformations with respect to  $\hat{\chi}$  and  $\hat{\lambda}$ .

Since all the preparations have been done, let us substitute the relation (3.11) and (3.15) for (3.7) to have the rewritten expression of the massive gauge transformation of  $\hat{A}^{(10)}$ :

$$\begin{aligned}\delta(i_{\hat{k}}\hat{A}^{(10)})_{\mu_1\cdots\mu_9} &= -9! \left[ \frac{1}{7!} \partial(i_{\hat{k}}\hat{N}^{(8)})\hat{\lambda} - \frac{1}{2 \cdot 5!} \partial\{(i_{\hat{k}}\hat{C}^{(6)})(i_{\hat{k}}\hat{C})\}\hat{\lambda} \right. \\ &\quad \left. + \frac{1}{6 \cdot 4!} \partial\{\hat{C}(i_{\hat{k}}\hat{C})^2\}\hat{\lambda} - \frac{\hat{M}}{2^4 \cdot 4!} (i_{\hat{k}}\hat{C})^4 \hat{\lambda} \right]_{[\mu_1\cdots\mu_9]}.\end{aligned}\quad (3.16)$$

By using this expression, the gauge invariant WZ action of the M-9-brane can be constructed. Before constructing it, we give the rewritten field equation of  $\hat{M}(x)$ :

$$\begin{aligned}-\sqrt{|\hat{g}|\hat{M}|\hat{k}|^4} &= \frac{10}{10!} \hat{\epsilon}^{\mu_1\cdots\mu_{10}z} \left\{ \partial_{\mu_1} (i_{\hat{k}}\hat{A}^{(10)})_{\mu_2\cdots\mu_{10}} - \frac{9!}{8 \cdot 6!} (i_{\hat{k}}\hat{G}^{(7)})(i_{\hat{k}}\hat{C})^2 + \frac{9!}{2 \cdot 8!} (i_{\hat{k}}\hat{G}^{(9)})(i_{\hat{k}}\hat{C}) \right. \\ &\quad \left. + \frac{9!}{288} \partial\hat{C}(i_{\hat{k}}C^{(3)})^3 + \frac{9 \cdot 9!}{144 \cdot 40} \hat{M}(i_{\hat{k}}C^{(3)})^5 \right\}_{\mu_1\cdots\mu_{10}}.\end{aligned}\quad (3.17)$$

Since the right hand side of (3.17) is shown to be gauge invariant, it can be interpreted as the gauge invariant field strength of the the 10-form (multiplied by  $1/10!$ ). Thus, we can conclude that the 10-form  $\hat{A}^{(10)}$  is introduced consistently. Moreover, we define a new 10-form  $\hat{C}^{(10)}$  which coincides with 10D IIA 9-form  $C^{(9)}$  on dimensional reduction along  $z$ :

$$\begin{aligned}(i_{\hat{k}}\hat{C}^{(10)})_{\mu_1\cdots\mu_9} &\equiv (i_{\hat{k}}\hat{A}^{(10)})_{\mu_1\cdots\mu_9} + 9! \left[ \frac{1}{2 \cdot 7!} (i_{\hat{k}}\hat{N}^{(8)})(i_{\hat{k}}\hat{C}) \right. \\ &\quad \left. - \frac{1}{2^3 \cdot 5!} (i_{\hat{k}}\hat{C}^{(6)})(i_{\hat{k}}\hat{C})^2 + \frac{1}{2^3 \cdot (3!)^2} \hat{C}(i_{\hat{k}}\hat{C})^3 \right]_{[\mu_1\cdots\mu_9]} \\ \hat{C}_{\mu_1\cdots\mu_{10}}^{(10)} &\equiv \hat{A}_{\mu_1\cdots\mu_{10}}^{(10)}.\end{aligned}\quad (3.18)$$

Then, the gauge transformation of  $\hat{C}^{(10)}$  takes the simple form:

$$\delta(i_{\hat{k}}\hat{C}^{(10)})_{\mu_1\cdots\mu_9} = -945M\{(i_{\hat{k}}\hat{C})^4\hat{\lambda}\}_{[\mu_1\cdots\mu_9]}.\quad (3.19)$$

For convenience, we use this 10-form to construct  $S_{M9}^{WZ}$ .

Finally, we construct the M-9-brane WZ action as that of the gauged  $\sigma$ -model, in which the translation along  $\hat{k}$  is gauged[23][10][11]. In this approach the M-9-brane wrapped around the compact isometry direction is described[13]. So, denoting its worldvolume coordinates by  $\xi^i$  ( $i = 0, 1, \dots, 8$ ) and their embeddings by  $X^\mu(\xi)$  ( $\mu = 0, 1, \dots, 9, z$ ), the worldvolume gauge transformation is given by

$$\delta_\eta X^\mu = \eta(\xi) \hat{k}^\mu \quad (3.20)$$

where  $\eta(\xi)$  is a scalar gauge parameter. In order to make the brane action invariant under the transformation, the derivative of  $X^\mu$  with respect to  $\xi^i$  is replaced by the covariant derivative[11]

$$D_i X^\mu = \partial_i x^\mu - \hat{A}_i \hat{k}^\mu \quad (3.21)$$

with the gauge field  $\hat{A}_i = -|\hat{k}|^{-2}\partial_i X^\nu \hat{k}_\nu$ . The dimensional reduction of  $D_i X^\mu$  is such that  $D_i X^\mu = \partial_i X^\mu$  for  $\mu \neq z$  and  $D_i X^z = -\partial_i X^\mu C_\mu^{(1)}$ . ( $\partial_i X^z = 0$  since  $X^z$  corresponds to  $z$ .) The M-9-brane action must be so constructed as to give the D-8-brane action on dimensional reduction along  $z$ . So, considering the field relations given in the appendix, we obtain the M-9-brane WZ action for a constant mass background  $\hat{M}(x) = m$ :

$$\begin{aligned}
S_{M9}^{WZ} = & T_{M9} \int d^9 \xi \epsilon^{i_1 \dots i_9} \left[ \frac{1}{9!} (\hat{k} \widetilde{\hat{C}}^{(10)})_{i_1 \dots i_9} + \frac{1}{7!} (\hat{k} \widetilde{\hat{N}}^{(8)})_{i_1 \dots i_7} \hat{\mathcal{K}}_{i_8 i_9}^{(2)} \right. \\
& + \frac{1}{2 \cdot 5!} (\hat{k} \widetilde{\hat{C}}^{(6)})_{i_1 \dots i_5} (\hat{\mathcal{K}}^{(2)})_{i_6 \dots i_9}^2 + \frac{1}{36} \tilde{\hat{C}}_{i_1 i_2 i_3} \{ (\partial \hat{b})^2 - \frac{1}{4} (\hat{k} \widetilde{\hat{C}}) \partial \hat{b} + \frac{1}{8} (\hat{k} \widetilde{\hat{C}})^2 \}_{i_4 \dots i_7} \hat{\mathcal{K}}_{i_8 i_9}^{(2)} \\
& \left. + \frac{1}{4!} \hat{A}_{i_1} \{ \partial \hat{b} + \frac{3}{2} (\hat{k} \widetilde{\hat{C}}) \}_{i_2 i_3} (\hat{\mathcal{K}}^{(2)})_{i_4 \dots i_9}^3 + \frac{m}{5!} \hat{b}_{i_1} (\partial \hat{b})_{i_2 \dots i_9}^4 \right] \quad (3.22)
\end{aligned}$$

where  $\tilde{\hat{S}}_{i_1 \dots i_r} \equiv \hat{S}_{\mu_1 \dots \mu_r} D_{i_1} x^{\mu_1} \dots D_{i_r} x^{\mu_r}$  for a target-space field  $\hat{S}_{\mu_1 \dots \mu_r}$ .  $\hat{b}_i$  describes the flux of an M-2-brane wrapped around the isometry direction, whose massive gauge transformation is determined by the requirement of the invariance of its modified field strength  $\hat{\mathcal{K}}_{ij}^{(2)} = 2\partial_{[i} \hat{b}_{j]} - \partial_i X^\mu \partial_j X^\nu (\hat{k} \widetilde{\hat{C}})_{\mu\nu}$ . We can see that this action is invariant (up to total derivative) under both the massive and the worldvolume gauge transformations (3.20). If one consider a worldvolume 8-form  $\hat{\omega}^{(8)}$  and add the term  $\int 9d\hat{\omega}^{(8)}$  to (3.22), the total derivative can be compensated and the action becomes exactly invariant. For later use, we also present the kinetic part of the M-9-brane action[15]:

$$S = -T_{M9} \int d^9 \xi |\hat{k}|^3 \sqrt{|\det(D_i X^\mu D_j X^\nu \hat{g}^{\mu\nu} + |\hat{k}|^{-1} \hat{\mathcal{K}}_{ij}^{(2)})|}. \quad (3.23)$$

## 4 A target-space M-9-brane solution

In this section we obtain an M-9-brane solution with a nontrivial 10-form, by solving the equations of motion with the source terms (i.e.  $\delta S^{\text{total}} \equiv \delta\{S_{\text{massive SUGRA}} + S_{M9}\} = 0$ ).

We set the ansatz for the target-space fields:

$$\begin{aligned}
ds^2 &= H^\alpha (dt^2 - dx_{(8)}^2) - H^\beta dy^2 - H^\gamma dz^2 \\
\hat{A}_{01, \dots, 8z}^{(10)} &= \hat{A}_{01, \dots, 8z}^{(10)}(y)
\end{aligned} \quad (4.1)$$

with all the other fields vanishing, and for the worldvolume fields:

$$X^i = \xi^i \quad \text{for } i = 0, 1, \dots, 8, \quad \text{and } \hat{b}_j = 0. \quad (4.2)$$

$H$  is a function depending on the single transverse direction  $y$ . Then, the nontrivial equations of motion are only those for  $\hat{g}_{\mu\nu}$ ,  $\hat{A}^{(10)}$ ,  $\hat{M}$ , and  $X^i$ .

Suppose the single M-9-brane we consider lies at  $y = 0$ . Then, the  $\hat{A}^{(10)}$  equation is<sup>¶</sup>

$$\partial_y \hat{M} = -T_{M9} \bar{\kappa} \delta(y), \quad \partial_\mu \hat{M} = 0 \quad \text{for } \mu \neq y \quad (4.3)$$

where  $1/\bar{\kappa} \equiv \int dx^z/\kappa$ . If we take the symmetry between the region  $y > 0$  and  $y < 0$  into account, it is proper to take the solution

$$\hat{M}(x) = \begin{cases} -\bar{m} & \text{for } y > 0 \\ \bar{m} & \text{for } y < 0 \end{cases}$$

where  $\bar{m} \equiv T_{M9} \tilde{\kappa}/2$ . In other words, the mass parameter is determined by the background M-9-brane as the above. Then, the Einstein equation parts of the field equations of  $\hat{g}_{\mu\nu}$  are the same as those before introducing the 10-form. So, we have  $\alpha = -\epsilon/3$ ,  $\beta = -10\epsilon/3 - 2$  and  $\gamma = 5\epsilon/3$  for a nonzero parameter  $\epsilon$ , as given in ref.[13]. Then, the field equations of  $\hat{g}_{\mu\nu}$  are

$$\frac{\delta S^{\text{total}}}{\delta \hat{g}_{\mu\nu}} = \frac{1}{2\bar{\kappa}} H^{2\epsilon/3} \eta_{\mu\nu} [\epsilon \partial_y^2 H + \frac{1}{2} H^{-1} \{\epsilon^2 (\partial_y H)^2 - \hat{M}^2\}] + \frac{T_{M9}}{2} H^{2\epsilon/3} \eta_{\mu\nu} \delta(y) = 0 \quad (4.4)$$

$$\frac{\delta S^{\text{total}}}{\delta \hat{g}_{yy}} = -\frac{1}{4\bar{\kappa}} H^{-7\epsilon/3-1} \{\epsilon^2 (\partial_y H)^2 - \hat{M}^2\} = 0 \quad (4.5)$$

$$\frac{\delta S^{\text{total}}}{\delta \hat{g}_{zz}} = -\frac{3}{2\bar{\kappa}} H^{8\epsilon/3} [\epsilon \partial_y^2 H + \frac{5}{3} H^{-1} \{\epsilon^2 (\partial_y H)^2 - \hat{M}^2\}] - \frac{3 \cdot T_{M9}}{2} H^{8\epsilon/3} \delta(y) = 0. \quad (4.6)$$

So, a solution to these equations is obtained as

$$H = c - \frac{T_{M9} \bar{\kappa}}{2\epsilon} |y| \quad (= c - \frac{\bar{m}}{\epsilon} |y|) \quad (4.7)$$

for an arbitrary nonzero  $\epsilon$  and a constant  $c$ . We note that  $H$  is a harmonic function on  $y$ . We also note that if one require  $H$  to be positive in order to avoid a singularity at  $H = 0$ , one must take  $c$  to be positive and  $\epsilon$  to be negative.

The remaining target-space field equation is that of  $M$ , which in this case is

$$\sqrt{|\hat{g}|} \hat{M} |\hat{k}|^4 + \frac{\hat{\epsilon}^{\mu_1 \dots \mu_{10} z}}{\sqrt{|\hat{g}|}} \frac{10}{10!} \partial_{[\mu_1} (i_{\hat{k}} \hat{A}^{(10)})_{\mu_2 \dots \mu_{10}]} = H^{\epsilon-1} \hat{M} - \partial_y \hat{A}_{01\dots 8z}^{(10)} = 0. \quad (4.8)$$

This equation determines (the field strength of)  $\hat{A}^{(10)}$ . It is solved by

$$\hat{A}_{01\dots 8z}^{(10)} = H^\epsilon. \quad (4.9)$$

Then,  $X^\mu$  field equations are satisfied. Thus, we can obtain the M-9-solution with a non-trivial 10-form by solving the equations of motion with the source terms. The dimensional reduction of the solution along  $z$  gives the D-8-brane solution[5].

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<sup>¶</sup>We note that because of the existence of the isometry direction,  $\frac{\delta L(x')}{\delta L(x)} = \delta^{(10)}(x - x')$  for a field  $L$ , instead of  $\delta^{(11)}(x - x')$ . Moreover, the integration with respect to  $x^z = z$  in the supergravity action is performed at the beginning.

## 5 Summary and discussion

We have constructed the Wess-Zumino action of a single M-9-brane wrapped around the compact direction. We have also obtained the M-9-solution with a nontrivial configuration of the 10-form by solving the equations of motion with the source terms. This implies the consistency of the M-9-brane action, not only the WZ term obtained in this paper but also the scaling factor  $|\hat{k}|^3$  of the M-9-brane kinetic action [13][15] (to be concrete, see (4.6)). Another consistency check can be done by considering a “test” M-9-brane in an M-9-brane background. The “static potential”  $V$  of the test M-9-brane is obtained by substituting the M-9-brane solution obtained in section 4 for the target-space fields of the M-9-brane effective action, as done in ref.[24]. Then, putting the test brane parallel to the background M-9-brane with an orientation, we have the potential  $V = H^\epsilon - H^\epsilon = 0$ . This implies that the test brane is stable, which is a reasonable result. So, we conclude that the introduction of the 10-form and the construction of the M-9-brane WZ action is achieved in a consistent way.

Furthermore, as discussed in ref.[25][15][17][16], there are other possibilities of dimensional reduction: The M-9-brane dimensionally reduced along the worldvolume direction but not the isometry direction is supposed to give an 8-brane with a gauged direction, called a “KK-8A brane. On the other hand, the M-9-brane dimensionally reduced along the transverse direction is to give an 9-brane called an “NS-9A brane”. The worldvolume actions of the former is given fully in ref.[16] and that of the latter is partly presented in ref.[17], but *both are obtained via T-duality*. So, deriving these actions *via dimensional reduction* from the M-9-brane action, including the WZ part obtained in this paper, will reinforce the consistency of the action. This appears to be possible, but we do not discuss this further here.

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## Appendix

In this appendix we present the relations between the 11D and the 10D fields via dimensional reduction in the isometry direction[10]. (Hats on indices indicate that they are 11-dimensional, and absence of Hats indicates that they are 10-dimensional.) The elfbein basis is

$$(\hat{e}_{\hat{\mu}}^{\hat{a}}) = \begin{pmatrix} e^{-\phi/3} e_{\mu}^a & e^{2\phi/3} C_{\mu}^{(1)} \\ 0 & e^{2\phi/3} \end{pmatrix}, \quad (\hat{e}_{\hat{a}}^{\hat{\mu}}) = \begin{pmatrix} e^{\phi/3} e_a^{\mu} & -e^{\phi/3} C_a^{(1)} \\ 0 & e^{-2\phi/3} \end{pmatrix}.$$

The 11D metric and the 11D 3-form gauge field are expressed as

$$\left\{ \begin{array}{l} \hat{g}_{\mu\nu} = e^{-2\phi/3} g_{\mu\nu} - e^{4\phi/3} C_{\mu}^{(1)} C_{\nu}^{(1)}, \\ \hat{g}_{\mu z} = (i_{\hat{k}} \hat{g})_{\mu} = -e^{4\phi/3} C_{\mu}^{(1)} \\ \hat{g}_{zz} = -e^{4\phi/3} \end{array} \right. \quad \left\{ \begin{array}{l} \hat{C}_{\mu\nu\rho} = C_{\mu\nu\rho}^{(3)} \\ \hat{C}_{\mu\nu z} = (i_{\hat{k}} \hat{C})_{\mu\nu} = B_{\mu\nu} \end{array} \right.$$

The 11D 6-form gauge field splits as

$$\left\{ \begin{array}{l} \hat{C}_{\mu_1 \dots \mu_6}^{(6)} = -\tilde{B}_{\mu_1 \dots \mu_6}^{(6)}, \\ \hat{C}_{\mu_1 \dots \mu_5 z}^{(6)} = (i_{\hat{k}} \hat{C}^{(6)})_{\mu_1 \dots \mu_5} = C_{\mu_1 \dots \mu_5}^{(5)} - 5C_{[\mu_1 \mu_2 \mu_3}^{(3)} B_{\mu_4 \mu_5]}, \end{array} \right.$$

where  $\tilde{B}^{(6)}$  is the 6-form field dual of  $B$ . The 11D 8-form gauge field is considered to give the 10D RR 7-form

$$(i_{\hat{k}} \hat{N}^{(8)})_{\mu_1 \dots \mu_7} = C_{\mu_1 \dots \mu_7}^{(7)} - 5C_{[\mu_1 \mu_2 \mu_3}^{(3)} B_{\mu_4 \mu_5} B_{\mu_6 \mu_7]},$$

and an 8-form. The 10D IIA 8-form seems to correspond to the dual of the dilaton. The 11D 2-form gauge parameter  $\hat{\chi}_{\mu\nu}$  corresponds to the RR 2-form gauge transformation parameter, while and  $\hat{\chi}_{\mu z} \equiv \hat{\lambda}_{\mu}$  to  $\lambda_{\mu}$ . We note that  $\hat{\epsilon}^{\mu_1 \dots \mu_{10} z} = \epsilon^{\mu_1 \dots \mu_{10}}$ , but  $\hat{\epsilon}_{\mu_1 \dots \mu_{10} z} = -\epsilon_{\mu_1 \dots \mu_{10}}$ .

Finally, the worldvolume gauge field of the M-9-brane  $\hat{b}_i$  gives that of the 10D IIA D-8-brane.

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